Ultrasound Tunneling through 3D Phononic Crystals

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(Received 22 August 2001; published 26 February 2002)

We report the study of ultrasound tunneling in 3D phononic crystals, consisting of fcc arrays of close-packed tungsten carbide beads in water. The transmission coefficient, phase velocity, and group velocity were measured along the [111] direction, allowing us to systematically investigate the tunneling of ultrasound at frequencies in the lowest band gap. Our experimental data are interpreted using multiple scattering theory, which provides a good explanation of our results. The effect of absorption and the difference between the tunneling of classical waves and quantum waves are discussed.

DOI: 10.1103/PhysRevLett.88.104301 PACS numbers: 43.35.+d, 63.20.Dj

The propagation of acoustic and elastic waves in periodic composite materials has been a subject of considerable interest in the past decade [1–13]. Because of the periodicity of the structure, there can be frequency ranges in which waves cannot propagate, giving rise to phononic band gaps which are analogous to photonic band gaps [14–16] for electromagnetic waves. Interest in phononic crystals comes from the rich physics of acoustic and elastic systems, where both the density and velocity contrast affect wave scattering and propagation, and where the waves can have mixed longitudinal and transverse vector properties. Potential applications of phononic crystals, e.g., in noise-proof devices and sound filters, have also attracted much recent interest [11,13]. The study of phononic crystals along with photonic crystals also furthers the study of wave phenomena such as localization [7,14–16].

Compared with theoretical work (e.g., [1–3,8–10]), there have been relatively few experimental investigations [4–7,10–13] of acoustic and elastic band gap materials. Most of the previous experimental studies have focused on 2D structures [4–7,13], and on measuring the transmission coefficient. While this is useful in identifying the band gaps, such measurements do not provide phase information, which is important for the study of wave propagation dynamics in phononic crystals. In this Letter, we examine wave transport through 3D phononic crystals both experimentally and theoretically, using a pulsed ultrasound technique and the multiple scattering theory (MST) [10]. Inside the phononic band gap, we demonstrate for the first time that ultrasound pulses tunnel through the crystals. The interesting effects of absorption are also examined. A simple model based on the cutoff of long scattering paths is shown to give a good account of the measured data.

Tunneling is one of the most striking phenomena in quantum mechanics. Recently, tunneling has also been seen in photonic crystals [17–19]. The tunneling time was found to be independent of the sample thickness, as predicted earlier for electron tunneling through a barrier [20,21]. The observed tunneling time was less than the time needed for a photon to travel the same distance in vacuum, implying a superluminal speed (although causality is not violated) [17–19]. Should one expect a similar behavior in phononic crystals?

To answer this question, we first calculated the band structure for a face-centered-cubic (fcc) crystal of tungsten carbide beads in water (Fig. 1), using the newly developed MST [10] that is ideally suited to calculating the band structure of phononic crystals consisting of solid spherical scatterers immersed in a fluid matrix. A large complete gap is found extending from dimensionless frequency 0.75 to 0.91 (corresponding to 0.98 to 1.2 MHz for our phononic crystals, which contain close-packed, 0.800-mm-diam beads). This complete gap is much wider than the one reported previously [12]. The widest part of the gap, extending from 0.8 to 1.2 MHz, is along the [111] direction, from Γ to L. Using a layer MST, we can also calculate the transmitted field for a multilayer sample with

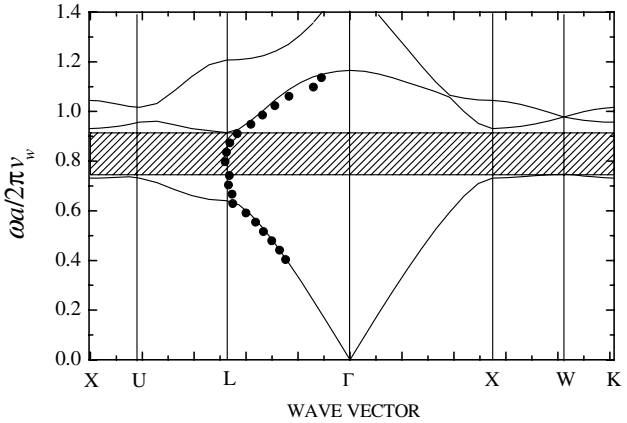

FIG. 1. Band structure of a fcc crystal of tungsten carbide beads immersed in water. Here a is the lattice constant and v_w is the sound velocity in water. The solid curves represent the theoretical predictions and the dots denote the experimental data. The shaded region shows the large complete gap.

thickness L. The transmitted wave can be written as

$$T(L,\omega) = A(L,\omega)\exp[i\phi(L,\omega)], \quad (1)$$

where A is the amplitude normalized by the input pulse, ϕ is the phase difference between the input and the transmitted pulses, and ω and k are the angular frequency and wave vector. From the phase difference, we determine the phase (ν_p) and group (ν_g) velocities:

$$\nu_p(\omega) = \frac{\omega}{k} - L\frac{\omega}{\phi}, \qquad \nu_g(\omega) = \frac{d\omega}{dk} = L\frac{d\omega}{d\phi}. \quad (2)$$

The advantage of the layer MST is that it provides a direct comparison with the measured wave field transmitted through the sample slab using our pulsed ultrasound technique.

The tungsten carbide beads provide a large scattering contrast with water, due to the large differences in both the density and the velocity of the two components (for tungsten carbide, the longitudinal and shear velocities are $\nu_L = 6.655$ km/s and $\nu_S = 3.23$ km/s, and the density is $\rho = 13.8 \times 10^3$ kg/m^3, while for water $\nu_L = 1.49$ km/s and $\rho = 1.0 \times 10^3$ kg/m^3). The beads are monodisperse and were assembled in a fcc crystal structure, arranged with the beads packed in triangular layers perpendicular to the body diagonal, or along the [111] direction. By carefully designing the sample template, made of acrylite, and putting the beads in by hand, we obtained very high quality crystals due to the high degree of monodispersity of the beads. The sample was placed horizontally on top of a substrate, which was made sufficiently thick so that multiple reflections in the substrate did not interfere with the sample signals. The ultrasonic waves transmitted through the sample were measured by placing the sample between two transducers and immersing the whole system in a large water tank. A $\frac{1}{2}$-inch-diam generating transducer was placed far away from the sample to yield an input pulse that was a plane wave to a good approximation. The transmitted pulses detected by the receiving transducer were amplified with a low-noise bandpass amplifier, averaged with a digital oscilloscope, and then downloaded to a computer for further analysis. The input pulse was determined by measuring the pulse transmitted through the substrate alone, without the sample.

Typical input and transmitted pulses through a six-layer phononic crystal and their Fourier spectra are shown in Fig. 2. Because of strong scattering inside the crystal, the transmitted pulse extends over a much longer time interval than the short input pulse, shown in Fig. 2(a). The amplitude of the transmitted pulse is about 12 times smaller. To measure the frequency dependence of the transmitted amplitude, we determine the transmission coefficient from the amplitudes of the transmitted and input signals at each frequency using a Fourier transform technique: $|T(\omega)| = A_{\text{trans}}(\omega)/A_{\text{in}}(\omega)$. The input pulse is essentially Gaussian with a central frequency of 1.0 MHz as shown

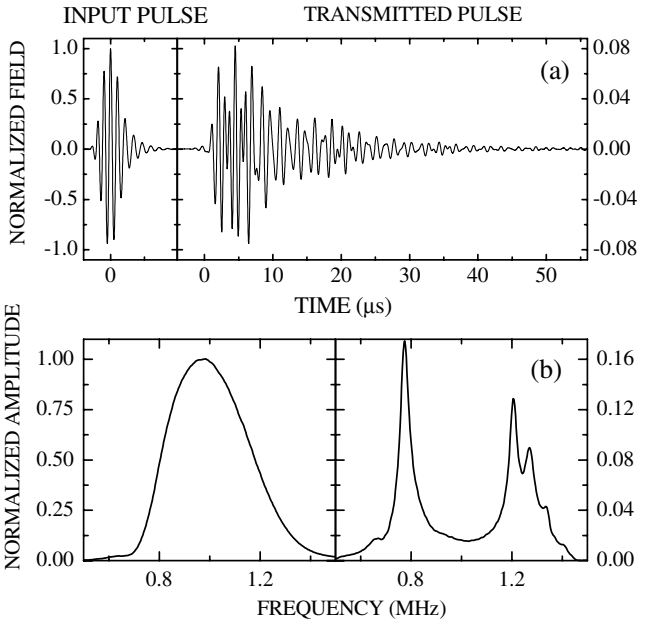

FIG. 2. (a) Transmitted ultrasonic pulse through a six-layer sample (right) compared with the input pulse (left). (b) The Fourier spectra of the two pulses in (a).

in Fig. 2(b), while the transmitted pulse shows a deep dip in this frequency range. This suggests that there is a gap around 1.0 MHz. To obtain a clearer picture, we have plotted the amplitude transmission coefficient in Fig. 3(a), and compared our data with theoretical predictions [10]. Good agreement is observed. Between 0.8 and 1.2 MHz, the transmission is very small along the [111] direction, and the dip in the transmission is deeper for the thicker sample. This agrees well with the gap shown in the band structure along the direction from Γ to L (Fig. 1).

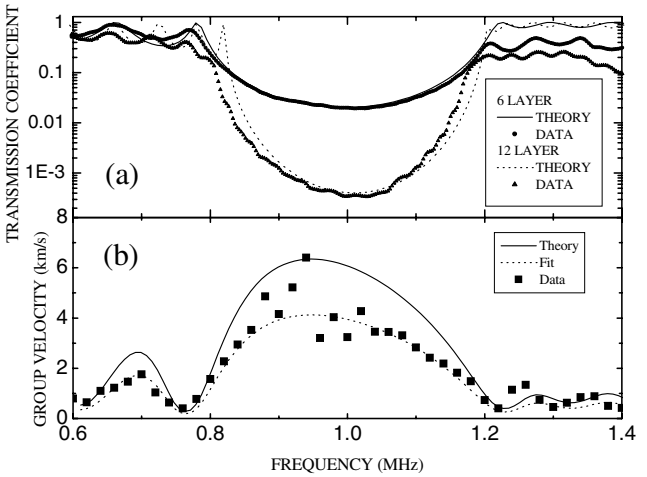

FIG. 3. (a) The theoretical and experimental frequency dependencies of the amplitude transmission coefficient for a six-layer and a twelve-layer sample, respectively. (b) The frequency dependence of the group velocity for a five-layer sample, where the curve denoted "fit" is the theoretical prediction scaled to fit the data.

The band structure in Fig. 1 also shows that, for our system, there is a complete gap around 1.0 MHz. Thus waves around 1.0 MHz cannot propagate *along any direction*. As a result, if there is a wave source inside the sample, the wave will be localized ("trapped") around the source. Yet, despite the breakdown of wave propagation in the gap, there is still a small signal transmitted through our samples.

To show that the ultrasonic pulse is transmitted by tunneling, we measure the group velocity by digitally filtering the input and transmitted pulses using a narrow Gaussian bandwidth (0.01 MHz). By measuring the time delay Δt_{peak} for the peak of a Gaussian pulse to travel through the sample, we experimentally determine the group velocity as $\nu_g(\omega) = L/\Delta t_{\text{peak}}$. Figure 3(b) shows the frequency dependence of the group velocity for a five-layer sample. The curve denoted "fit" is the theory curve multiplied by a scale factor to fit the data. Variation by more than a factor of 10 is observed in the group velocity over the frequency range in and around the gap. At the frequency of 0.945 MHz, the group velocity reaches it maximum. Because of absorption (see below), the measured values are smaller than the theoretical predictions. However, the basic trend is well captured by the theory.

The group velocity in the gap increases monotonically with the sample thickness, as shown in Fig. 4 at a frequency of 0.945 MHz. In particular, the group velocity is very large for thick samples, where both experimental and theoretical values can be bigger than the longitudinal velocity in either component (water or tungsten carbide). In addition, we have shown that these large values of the group velocity are not affected by pressure applied to the beads [22]. Our results demonstrate convincingly that tunneling is involved, since normally the velocity is independent of the distance traveled, while for tunneling it is the transit time that is essentially independent of the sample thickness, leading to the observed increase in ν_g with L. To understand the effect of absorption, we include absorption rigorously for a simple model 1D phononic crystal. The results are shown in the inset of Fig. 4. We see clearly that absorption makes the group velocity smaller. This effect can be understood in simple physical terms as follows. Absorption reduces the contributions to Bragg-scattered waves from long multiply reflected paths, so that the destructive interference (that gives rise to a band gap) becomes incomplete. Thus the tunneling of a wave pulse through the gap becomes modified, and the wave pulse behaves as though there is a small additional propagating component with an effective wave vector determined by the incomplete cancellation of the Bragg-scattered waves. This has the net effect of increasing the time of propagation. By approximating the pulse transport as a dominant tunneling process in parallel with a small propagating component, we are able to accurately characterize the effect of absorption on the group velocity for the rigorous 1D model as well as for our 3D crystal (see the dashed curves in Fig. 4). This simple picture suggests an intriguing paradox: Inside a phononic band gap, the effect of absorption, itself characterized by an imaginary wave vector, appears to modify the tunneling of the wave pulse so as to introduce a small component having a real wave vector.

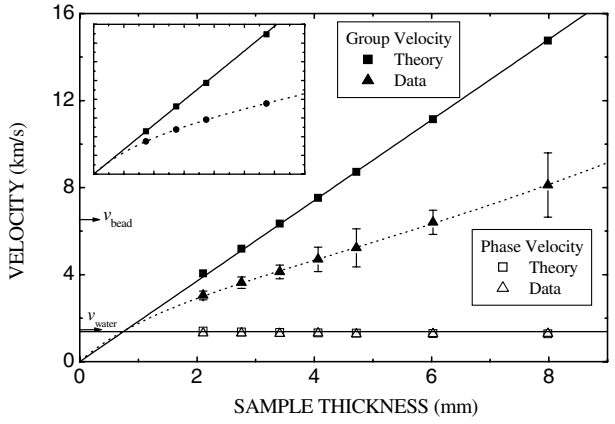

FIG. 4. The group velocity and phase velocity at 0.945 MHz versus sample thickness. The two arrows indicate the longitudinal velocities in tungsten carbide and water, respectively. Inset: The theoretical prediction of group velocity versus sample thickness for a 1D phononic crystal with (circles) and without (squares) absorption. Solid lines are a guide to the eye and dotted curves are fits to the simple two-mode model.

In Fig. 4, we also plot the phase velocity (open symbols), which is seen to be almost independent of sample thickness. The data are in very good agreement with the MST predictions. The comparison between experiment and theory also shows that absorption has almost no effect on the phase velocity, indicating that absorption has much less effect on the phase than it does on the derivative of the phase with frequency.

In order to see the difference between classical wave tunneling in phononic crystals and quantum particle tunneling, it is informative to consider electron tunneling through a potential barrier with height V_0 and thickness L [20]. By using the proper boundary conditions for the wave function, the phase difference ϕ between the transmitted and the input waves can be expressed as

$$\phi = \arctan\left[\frac{(2\hbar\omega - V_0)\tanh(L/2l)}{2\sqrt{(V_0 - \hbar\omega)\hbar\omega}}\right], \qquad (3)$$

where $l = \hbar/\sqrt{8m(V_0 - \hbar\omega)}$ is the tunneling length, defined by $|T|^2 = \exp(-L/l)$. As we can see from Eq. (3), when the sample is sufficiently thick, $\tanh(L/2l)$ goes to 1, so that the phase difference is independent of sample thickness for $L \gg l$. Thus the phase time $t_p = \phi/\omega$ and tunneling time of a pulse $t_g = d\phi/d\omega$ are both independent of sample thickness for large L, so that both the phase velocity ν_p and the group velocity ν_g increase linearly with sample thickness [see Eq. (2)]. Hence, whereas the group velocity increases linearly with sample thickness for both the quantum and the classical waves, for the phase velocity there is a qualitative difference. This difference pre-

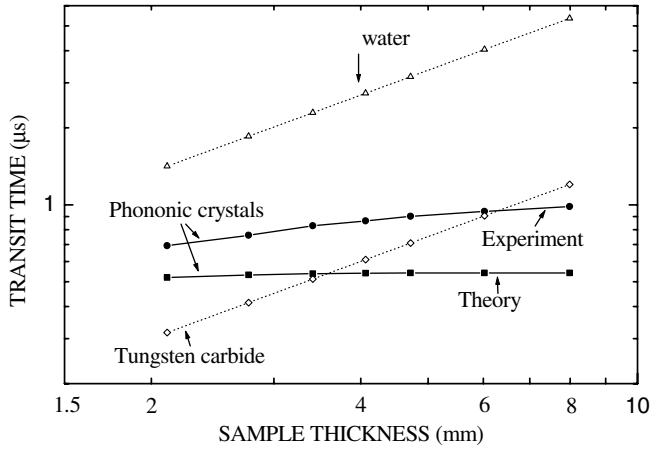

FIG. 5. Tunneling time versus sample thickness (solid symbols and lines) compared with the transit time (through the same distance) in water or tungsten carbide (open symbols with dotted lines).

sumably reflects the different origins of the gap in the two cases; for classical waves the gap arises from the cancellation of Bragg scatterings, while for the electrons it is due to the potential barrier.

In Fig. 5, we plot the tunneling time directly versus the sample thickness. Even though the experimental tunneling time is bigger than the MST prediction due to absorption, both times are approximately independent of sample thickness, each approaching a constant value at large L. By contrast, for normally propagating waves (i.e., away from the gap frequencies, or in pure materials such as water or tungsten carbide), the transit time increases linearly with distance. Therefore, Fig. 5 clearly shows that ultrasound tunnels through the phononic crystals at frequencies inside the gap. The magnitude of the tunneling time is remarkably short, with the tunneling time t_{tun} being approximately inversely proportional to the width of the gap, $\Delta\omega_{\text{gap}}$, so that in the middle of the gap $t_{\text{tun}}\Delta\omega_{\text{gap}} \sim 1$. Thus it is not surprising that for photon tunneling [17,18] the tunneling time is about 9 orders of magnitude shorter, reflecting the much higher frequencies involved.

We have presented both theoretical and experimental results for the transport of ultrasonic waves through fcc phononic crystals consisting of tungsten carbide beads in water. Our theoretical results show that there is a complete band gap in this system. Inside the gap, the group velocity increases with the sample thickness, demonstrating that tunneling is involved. Our data are well explained by the multiple scattering theory, which gives good overall agreement with the experiments.

Support from NSERC of Canada and RGC (HKUST-6143/00P) of Hong Kong is gratefully acknowledged.

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[22] In these experiments, the pressure pushing the beads together was increased so as to change the contact forces between the beads by an amount exceeding the buoyant weight of the beads, as well as the viscous forces exerted on the beads by the ultrasonic displacements in the water. Thus our tunneling data are not affected by ultrasonic propagation along the bead network, or by wave transport via coupled oscillations of the beads. The reason that the group velocity is independent of the applied pressure can be obtained from the multiple scattering theory, which predicts that the ultrasonic energy density is extremely small inside the beads at frequencies in the band gap (less than 1% of the energy density in the water).